Calculus Warm Up #3-3

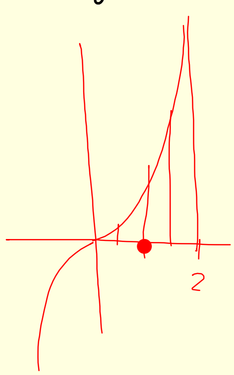
Use geometry to find the area between the curve and the x-axis represented by the following integrals. (No calculator)

1) $\int_1^5 3\,dx$ 2) $\int_{-2}^{2} \sqrt{4-x^2}\,dx$ 3) $\int_0^3 \frac{x^2-4}{x-2}\,dx$

HW Questions: p. 287

(trapezoids)

3) $\int_0^2 x^3\,dx$, $n=4$ 5) $\int_0^2 x^3\,dx$, $n=8$



$A = \frac{1}{2}(b_1 + b_2)h$

$h = \Delta x = \frac{b-a}{n}$

$A \approx \frac{1}{2}\overset{\Delta x}{\left(\frac{2-0}{4}\right)}\left[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + f(x_4)\right]$

$A \approx \frac{1}{4}\left[0^3 + 2\left(\frac{1}{2}\right)^3 + 2(1)^3 + 2\left(\frac{3}{2}\right)^3 + 2^3\right]$

$\approx \frac{1}{4}\left[0 + \frac{1}{4} + 2 + \frac{27}{4} + 8\right]$

$\approx \frac{17}{4}$

$x_0 = 0$

$x_1 = \frac{1}{2}$

$x_2 = 1$

$x_3 = \frac{3}{2}$

$x_4 = 2$

9) $\int_1^2 \frac{1}{(x+1)^2}\,dx$, $n=4$

HW Questions: p. 250 (midpoints)

57) $f(x)=x^2+3$ $[0,2]$

58) $f(x)=x^2+4x$ $[0,4]$

$\Delta x = \frac{4-0}{4} = 1$

Riemann Sums

1) For a continuous function, f(x), partitioned into subintervals of width Δx.

2) x_i is chosen on each subinterval (left or right endpoints, midpoints or arbitrary).

3) Rectangles drawn to the x-axis where h = $f(x_i)$

4) Areas of each rectangle are found, A = $[f(x_i)]\Delta x$ then summed, $\sum$

$$\sum_{i=1}^{n} [f(x_i)]\Delta x$$

The Definite Integral: The limit of a Riemann Sum

Definition of the Definite Integral

For f(x) defined on [a, b]

$$\lim_{\|\Delta\|\to 0} \sum_{i=1}^{n} \Delta x[f(x_i)] = \int_a^b f(x)\, dx$$

$\|\Delta\|$ is called the norm of the partition

a = lower limit of integration
b = upper limit of integration

the largest width Δx when they are not uniform.

The Definite Integral

For f(x) continuous on [a, b] $\int_a^b f(x)\, dx$

Gives us a number.

The Indefinite Integral $\int f(x)\, dx$

Gives us a family of curves

functions: F(x), the antiderivative of *f*

today

5.4 -The Fundamental Theorem of Calculus

-The Mean Value Theorem for Integrals

-Average value of a function on an interval

-The Second Fundamental Theorem of Calculus

tomorrow.

The Fundamental Theorem of Calculus

Essentially: Differentiation and Definite Integration are inverse operations.

If *f* is continuous on [a, b], and F'(x) = f(x)

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

The Fundamental Theorem of Calculus

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Examples:

from the warm-up

$$\int_1^5 3\,dx = \boxed{12} \rightarrow \int_1^5 3\,dx = \left[\underbrace{3x}_{F(x)}\right]_1^5$$

$$= 3(5) - 3(1)$$

$$= \boxed{12}$$

$$\int_1^3 x^3\,dx = \left[\frac{x^4}{4}\right]_1^3$$

$$= \frac{3^4}{4} - \frac{1^4}{4}$$

$$= \frac{81-1}{4}$$

$$= 20$$

Notice the constant of integration, C, can be dropped from the antiderivative:

$$\int_a^b f(x)\,dx = \Big[F(x) + C\Big]_a^b$$

$$= F(b) + C - \Big[F(a) + C\Big]$$

$$= F(b) - F(a)$$

EXAMPLE 1 Evaluating a definite integral

(a) $\int_1^2 (x^2 - 3)\,dx$

$$= \left[\frac{x^3}{3} - 3x\right]_1^2$$

$$= \frac{8}{3} - 6 - \left(\frac{1}{3} - 3\right)$$

$$= \frac{7}{3} - \frac{3}{1}\cdot\frac{3}{3}$$

$$= \boxed{-\frac{2}{3}}$$

(b) $\int_1^4 3\sqrt{x}\,dx$

$$= 3\int_1^4 x^{1/2}\,dx$$

$$= 3\left[\frac{2x^{3/2}}{3}\right]_1^4$$

$$= 2\left[x^{3/2}\right]_1^4$$

$$= 2\left((\sqrt{4})^3 - (\sqrt{1})^3\right)$$

$$= \boxed{14}$$

EXAMPLE 2 A definite integral involving absolute value

Evaluate

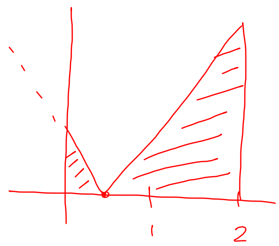
$$\int_0^2 |2x-1|\,dx. = \int_0^{1/2}(-2x+1)\,dx + \int_{1/2}^{2}(2x-1)\,dx$$

$$= \Big[-x^2+x\Big]_0^{1/2} + \Big[x^2-x\Big]_{1/2}^{2}$$

$$= -\frac{1}{4} + \frac{1}{2} - 0 + \underbrace{4-2}_{} - \left(\frac{1}{4} - \frac{1}{2}\right)$$

$$= -\frac{1}{2} + 1 + 2$$

$$= \boxed{\frac{5}{2}}$$



$$|2x-1| = \begin{cases} 2x-1 & ; x > \frac{1}{2} \\ -(2x-1) & ; x \le \frac{1}{2} \end{cases}$$

HW:

p. 267, # 1 - 25 odd

HW Quiz Friday:

pgs. 238, 248, 249, 258, and

trapezoids p. 287 / midpoints p. 250